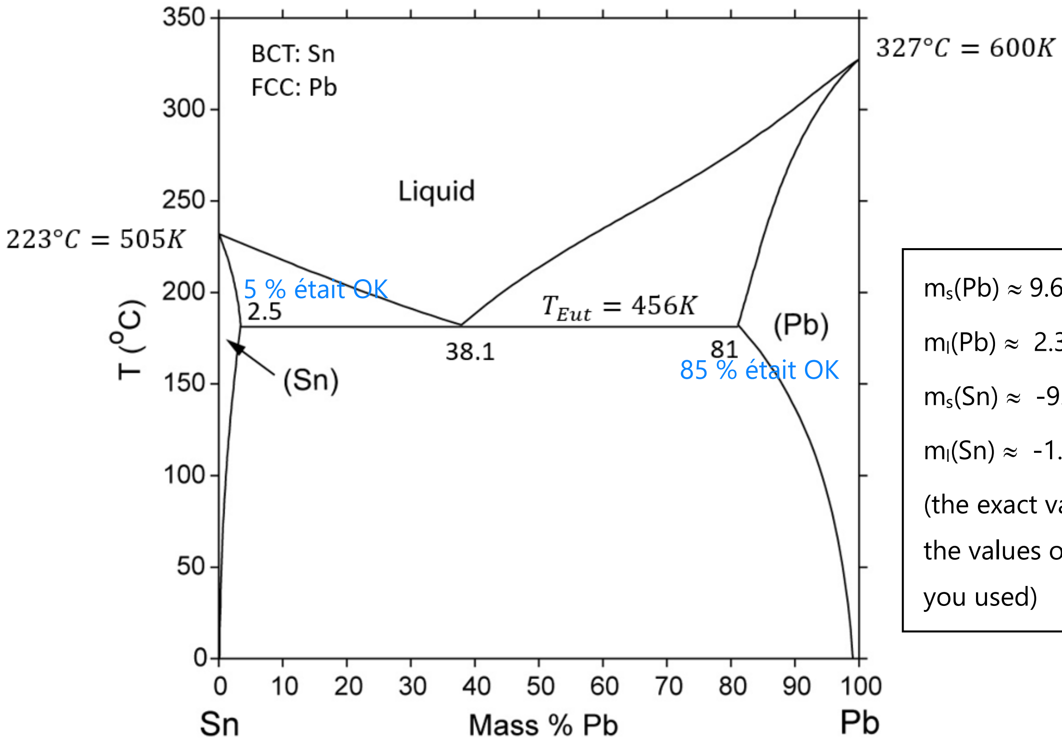
Solutions

Partie 1: voir cours

Partie 2. Solutions

Ex 2.1 solutions



$m_s(Pb) \approx 9.6$ K/wt%

$m_l(Pb) \approx 2.3$ K/wt%

$m_s(Sn) \approx -9.8$ K/wt%

$m_l(Sn) \approx -1.2$ K/wt%

(the exact values depend on the values of concentrations you used)

Ex 2.2 solutions

$$\Delta S_{m,Pb}^{s\to liq} = \frac{\Delta G_{m,Pb}^{s\to liq}}{\Delta T} = \frac{0-(-397)}{650-600}\ \frac{J}{mol\ K} = 7.94\ \frac{J}{mol\ K}$$

$$\Delta S_{m,Sn}^{s\to liq} = \frac{\Delta G_{m,Sn}^{s\to liq}}{\Delta T} = \frac{0-(-278)}{525-505}\ \frac{J}{mol\ K} = 13.9\ \frac{J}{mol\ K}$$

$$L_{m,Pb} = \Delta S_{m,Pb}^{s\to liq}\ T_{f,Pb} = 4764\ \frac{J}{mol}$$

$$L_{m,Sn} = \Delta S_{m,Sn}^{s\to liq}\ T_{f,Sn} = 7019.5\ \frac{J}{mol}$$

Ex 2.3 solutions

$$V_{m,Sn}^{s} = 16.555\ \frac{cm^3}{mol}$$

$$V_{m,Pb}^{s} = \frac{N_A}{4} a_{Pb}^3 = 18.25\ \frac{cm^3}{mol}$$

$$V_{m,Sn}^{liq} = \left(from\ Fig.2\ at\ T = T_{f,Sn}\right) = 0.1435\ \frac{cm^3}{g} = \ 0.1435 \frac{cm^3}{g} \times 118.7\ \frac{g}{mol} = 17.033\ \frac{cm^3}{mol}$$

$$\Delta V_{m,Pb}^{s\rightarrow liq} = \left(\frac{dT}{dp}\right)\bigg|_{eq} \frac{L_{m,Pb}}{T_{f,Pb}} = \ 1.12748\ \frac{cm^3}{mol}$$

$$\frac{\Delta V_{m,Sn}^{s\rightarrow liq}}{V_{m,Sn}^{s}} = \frac{17.033 - 16.555}{16.555} \times 100 = 2.88\%$$

$$\frac{\Delta V_{m,Pb}^{s\rightarrow liq}}{V_{m,Pb}^{s}} = \frac{1.12748}{18.25} \times 100 = 6.17\%$$

Ex 2.4 solutions

a)

$$\Delta G_n = \ -L^3 \Delta S_{f,Pb\rightarrow liq}^{v} \Delta T_n + 6L^2 \gamma_{sl}$$

b)

$$\frac{\partial \Delta G_n}{\partial L} = -3L^2 \Delta S_{f,Pb\rightarrow liq}^{v} \Delta T_n + 12L\ \gamma_{sl}$$

$$\frac{\partial \Delta G_n}{\partial L} = 0 \rightarrow\ L^2 \Delta S_f^v \Delta T_n = 4L\ \gamma_{sl} \rightarrow\ \Delta T_n = \frac{4\ \gamma_{sl}}{L\ \Delta S_{f,Pb\rightarrow liq}^{v}}$$

$$\Delta S_{f,Pb\rightarrow liq}^{v} = \ \frac{\Delta S_{m,Pb}^{s\rightarrow liq}}{V_{m,Pb}^{s}} = 435068\ \frac{J}{m^3 K}$$

$$\Delta T_n = \frac{4\ \gamma_{sl}}{L\ \Delta S_{f,Pb\rightarrow liq}^{v}} = \frac{4 \times 4499.39 \times 10^{-6}}{2 \times 10^{-9} \times 435068} = 20.68\ K$$

c)

$$T_{f,alloy} = 600K -\ m_{liq,B} \times (100 - 60)wt\% = 516\ K$$

$$\Delta T_n = 20.68\ K$$

$$T_1 = 500\ K >\ T_{f,alloy} -\ \Delta T_n$$

$$\rightarrow\ \ The\ system\ is\ consituted\ by\ the\ liquid\ phase\ as\ the\ solidification\ has\ not\ yet\ started$$

$$T_1 = 470\ K <\ T_{f,alloy} -\ \Delta T_n$$

$$\rightarrow\ \ The\ system\ is\ consituted\ by\ the\ liquid\ and\ Pb\ solid\ phases\ as\ the\ solidification\ has\ started$$

Ex 2.5 solutions

a) No. In steady state we would have $C_l^* = \frac{C_0}{k} > C_{eut.}$ → before reaching the steady state regime we cross the eutectic temperature T=456 K.

b) $\delta(at\ 456\ K) = \frac{D_{Pb}}{v} \times \frac{C_l^* - C_0}{C_l^* - C_s} = \frac{2.74\times10^{-10}}{0.5\times10^{-6}} \times \frac{61.9-40}{61.9-19} = 279\ \mu m$

Ex 2.6 solutions

Sn- 38.1 wt.% Pb is the eutectic composition: we have an eutectic microstructure

$$\alpha_{B,Sn} = \frac{\Delta S_{m,Sn}^{s\to liq}}{k_B \times N_A} = 0.954 < 2 \;; \quad \alpha_{B,Pb} = \frac{\Delta S_{m,Pb}^{s\to liq}}{k_B \times N_A} = 1.671 < 2$$

$$\to \; the\ morphology\ will\ be\ regular$$

$$g_\alpha = \frac{81 - 38.1}{81 - 2.5} = 0.546 \;; \quad g_\beta = \frac{38.1 - 2.5}{81 - 2.5} = 0.454 \;\to the\ morphology\ will\ be\ lamellar$$

<u>Partie 3. Solutions</u>

<u>Ex 3.1 solution</u> Σ=5

<u>Ex 3.2</u> solution ***p*** =(h,k) = (-1,2), ***d*** = -[2,1]

<u>Ex 3.3 solution</u>

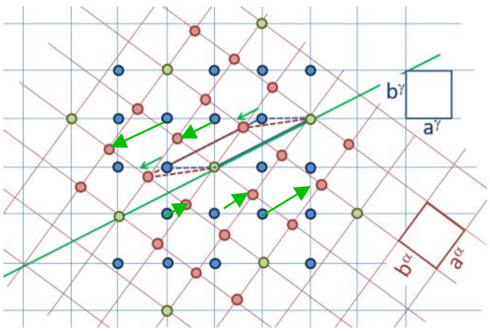
a) the mirror symmetry leaves invariant the direction **d** and reverses the normal of the plan **p**, We use the basis $\mathbf{B}_1$ made of **d**, and **n** the normal of the plane **p**, in a cubic lattice **n** = [-1,2].

$[B^\gamma \to B1] = \begin{pmatrix} 2 & -1 \\ 1 & 2 \end{pmatrix}$, $[B^\alpha \to B1] = \begin{pmatrix} 2 & 1 \\ 1 & -2 \end{pmatrix}$

M = $[B^\gamma \to B1][B^\alpha \to B1]^{-1} = \begin{pmatrix} 2 & -1 \\ 1 & 2 \end{pmatrix} \frac{1}{5} \begin{pmatrix} 2 & 1 \\ 1 & -2 \end{pmatrix} = \frac{1}{5}\begin{pmatrix} 3 & 4 \\ 4 & -3 \end{pmatrix}$

b) **R** = $\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \frac{1}{5}\begin{pmatrix} 3 & 4 \\ 4 & -3 \end{pmatrix} = \frac{1}{5}\begin{pmatrix} 3 & -4 \\ 4 & 3 \end{pmatrix}$

<u>Ex 3.4 solution</u>



$\mathbf{C}^{\alpha\to\gamma} = \begin{pmatrix} 1 & 0 \\ -1 & 1 \end{pmatrix}$

<u>Ex 3.5 solution</u>

a) $s^2 = \mathrm{Tr}(\mathbf{C}^t\mathbf{G}\,\mathbf{C}\,\mathbf{G}^{-1}) - 2$ with $\mathbf{G} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \to s = \mathrm{Tr}\begin{pmatrix} 1 & -1 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ -1 & 1 \end{pmatrix} - 2 = \mathrm{Tr}\begin{pmatrix} 2 & -1 \\ 1 & 1 \end{pmatrix} - 2 = 1$

b) $\mathbf{S} = \mathbf{I} + s\,\tilde{\mathbf{d}}.\tilde{\mathbf{p}}^t = \mathbf{I} + s\,\tilde{\mathbf{d}} \otimes \tilde{\mathbf{p}}$

$\to \mathbf{S} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} + 1\frac{1}{\sqrt{5}}\begin{pmatrix} -2 \\ -1 \end{pmatrix}\frac{1}{\sqrt{5}}\begin{pmatrix} -1 & 2 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} + \frac{1}{5}\begin{pmatrix} 2 & -4 \\ 1 & -2 \end{pmatrix} = \frac{1}{5}\begin{pmatrix} 7 & -4 \\ 1 & 3 \end{pmatrix}$

<u>Ex 3.6 solution</u> Utiliser la formule $\mathbf{C}^{\alpha\to\gamma} = \mathbf{T}^{\alpha\to\gamma}\mathbf{F}^\gamma$ with $\mathbf{F}^\gamma = \mathbf{S}$ and $\mathbf{T}^{\alpha\to\gamma} = \mathbf{R}^{-1}$